Estimating the Effects of Tunneling on Existing Pipelines

T. E. B. Vorster1; Assaf Klar2; Kenichi Soga3; and R. J. Mair4

Abstract: A method is presented for estimating the maximum bending moment for continuous (or rigidly jointed) pipelines affected by tunnel-induced ground movement. The estimation can be made based on the knowledge of tunnel and pipeline geometries, the stiffness of soil and pipeline, and tunnel-induced ground deformation at the pipeline level. The method takes account of soil nonlinearity by an equivalent linear approach, in which the stiffness of the soil is evaluated based on an average deviatoric strain developed along the pipeline. The approach is conservative and promises that the bending moment is not underestimated. The validity of the method as an upper bound approximation is evaluated against centrifuge test results.

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CE Database subject headings: Pipelines; Soil-pipe interaction; Tunneling; Settlement; Aging; Infrastructure; Design.

Introduction

One of the challenges facing engineers in the 21st century is operation and maintenance of aging infrastructure. Such infrastructure may include existing pipelines, which are frequently affected by third party activity, for instance tunneling. As can be deduced from case studies on small diameter pipelines affected by nearby tunneling (e.g., O’Rourke and Trautmann 1982; Owen 1987), it is evident that the problem of soil-pipeline-tunnel interaction is relatively complex. This problem was addressed in the past by Attewell et al. (1986), who presented a series of design graphs, which were obtained by approximating the problem to a Winkler system and solving it numerically. Klar et al. (2005a) provided a closed form solution for the Winkler problem of Attewell et al. (1986) and suggested an improved subgrade modulus value for the Winkler model so that the estimated maximum bending moments match the values calculated using a more rigorous continuum elastic solution.

Notwithstanding finite element analysis or formulating a Winkler-type model to evaluate the problem, designers may employ the method of Attewell et al. (1986). However, more frequently than not, the pipeline is assumed to follow the estimated green field settlement profile without taking account of the stiffness of the pipeline. This might lead to overestimation of bending moments. Although useful and relatively simple to use, these design methods do not provide engineers with an expectation of the realism of the design assumption. This paper aims to propose an alternative design method for continuous (or rigidly jointed) pipelines that recognizes key interaction factors. This results in a robust method offering an upper approximation of pipeline bending moment, which is still significantly lower than “forcing” the pipeline to follow the green field settlement profile. The method is convenient to use, utilizing design graphs and/or simple equations.

It will be shown that the solution may be highly sensitive to the green field displacement profile, depending on a relative pipe-soil-tunnel rigidity parameter defined in the current method. That means that even if one undertakes advanced finite element analyses which rigorously consider the pipe-soil-tunnel interaction, the solution may still deviate significantly from the true behavior simply because the calculated green field displacements may not be the same as the field displacements (PLAXIS 2001). To overcome this inherent problem, the proposed method suggests combining field measurements with analytical evaluation; that is, the method may be used to define guidelines to the extent of the field measurements and interaction analysis required.

Problem Definition

Fig. 1 shows a schematic of the problem, in which a new tunnel is excavated under an existing pipeline. The tunnel excavation generates soil settlement around the pipeline causing it to deform and suffer additional bending moment. The magnitude of pipeline deformation and the changes in bending moment depend on the distribution of soil settlement by tunneling at the pipeline level and the relative stiffness between the pipeline and the surrounding soil. The maximum bending moment occurs above the tunnel centerline, and is generally referred to as the sagging moment. Two extreme cases exist for the maximum bending moment and consequently the bending moment in the pipeline can only tend toward these cases, namely:

1. An infinitely flexible pipeline which follows the green field displacement profile perfectly, and
2. An extremely rigid pipeline which experiences the green field displacement as a localized disturbance. These two cases can easily be illustrated by means of a Winkler model and using the green field soil displacement profile. In the former case, the bending moment is simply equal to

\[ M(x) = -EI \frac{d^2 S}{dx^2} \]  

(1)

where \( M \) = bending moment; \( EI \) = pipeline bending stiffness; and \( S \) = green field settlement profile along a horizontal axis \( x \). A Gaussian curve, or a modified form of it, can be used for \( S \).

In the latter case, an expression for the sagging moment can be derived assuming a Winkler model

\[ M_{\text{max}} = \frac{P}{4} \left( \frac{2kr_0}{4EI} \right) = 0.6k^{0.75}r_0^{0.75}E(0.25)S \]

(2)

where \( r_0 \) = radius of the pipeline; \( VL \) = volume loss at the pipeline level; and \( k \) = coefficient of subgrade reaction (dimensions \( \text{FL}^{-3} \)), e.g., Attewell et al. (1986) and Klar et al. (2005a). This expression is based on the closed form solution for a point load (the localized disturbance) on a Winkler beam (Hetenyi 1946). In fact, Eq. (2) is the limit case of the Winkler solution of the general problem which can be obtained by superimposing solutions of infinite Winkler beams loaded by infinitely small loads distributed along the pipeline.

As the stiffness of the pipeline increases, \( \lambda \) decreases. When it approaches zero (infinitely stiff pipeline), Eq. (3) becomes Eq. (2) for the maximum moment at \( x=0 \).

**A Continuum Elastic Solution**

In the buildup to a useful design method, it is fitting to first consider a linear solution. For this purpose five key assumptions are made in defining the problem:

1. The tunnel is not affected by the presence of the pipeline.
2. The pipeline is continuous, elastic, and homogeneous and buried in homogeneous soil.
3. The soil response to loading, at levels of the pipeline, is unaware of the tunnel. This relaxing assumption allows us the use of Mindlin’s (1936) Green’s function for vertical load in a semi-infinite half space.
4. The green field soil displacement at the pipeline level is described by a general shape function.
5. The pipeline remains in contact with the soil.

Following these assumptions the pipeline behavior is represented by

\[ [S][u] = [F] \]

(4)

where \([S]\) is the stiffness matrix of the pipeline composed of standard beam elements, \([u]\) is the pipeline displacement, and \([F]\) is a force vector representing the soil loading. It is assumed that additional external loads to the soil do not generally exist, although they can be added if necessary.

The soil continuum displacement is represented using a Green’s function

\[ [u^{C}]_i = \sum_{j=1}^{n} [\{f\}_jG_{ij}] \]

(5)

where \( [\{f\}]_i \) = force acting on the soil medium; and \( G_{ij} \) = Green’s function which defines the elastic soil continuum displacement at point \( i \) due to unit loading at point \( j \). The Mindlin (1936) solution (Green’s function) for a point load is used. However, since Mindlin’s (1936) solution does not satisfy displacement at the point of loading, a reference displacement value for that point is considered to be the average displacement around the circumference of the pipeline. This is identical to assuming a barrel load around the pipeline as shown by Klar et al. (2005a), who provided a continuum elastic solution for the Gaussian case.

The summation in Eq. (5) can also be written as follows:

\[ [u^{CL}]_i = \sum_{j=1}^{n} [\{f\}_jG_{ij}]_{j \neq i} + [\{f\}_jG_{ij}]_{j = i} \]

(6)

where \([u^{CL}]_i \) is defined herein as local displacement, which is the displacement at a point solely due to its loading and...
\{u^{\text{CAP}}\} = \text{additional displacement at that point due to forces acting at different points. Due to assumption (1) only degrees of freedom of the pipeline need to be considered and the } i \text{ index can therefore be related only to the pipeline. Nevertheless, } \{u^{\text{CAT}}\} \text{ still involves quantities that result from the tunnel (i.e., the } j \text{ index is still related to the tunnel degree of freedom). Eq. (6) can further be decomposed as follows:}

\[ \{u\}_j = \{f\}_j G_{ij} + \sum_{j \neq i} \{f\}_j G_{ij} + \{u^{\text{CAT}}\}_j \]

where \{u^{\text{CAP}}\} = \text{additional displacement due to forces resulting from pipe-soil interaction and } \{u^{\text{CAT}}\} = \text{additional displacement due to the existence of the tunnel. By using assumption (1), } u^{\text{CAT}} \text{ is defined as the green field settlement profile at pipeline level. For very large diameter pipelines in close proximity to the tunnel the use of } u^{\text{CAT}} \text{ as the green field displacement may not be representative. In such cases the effect of the existence of the pipeline on the tunnel should be taken into account by means of a more sophisticated model.}

Remembering that the force acting on the soil is the reaction for the structure, one can define the soil reaction on the structure from Eq. (6)

\[ \{F\}_i = -\{f\}_i = -\{u^{\text{CL}}\}_i G_{ij} \]

The compatibility relation of \{u\} = \{u^{r}\} = \{u^{\text{CL}}\} + \{u^{\text{CAP}}\} + \{u^{\text{CAT}}\} \text{ is required, and by introducing this and Eq. (8) into Eq. (4) the following relation can be obtained:}

\[ [S][u] + [K^*][u] = [K^*][u^{\text{CAP}}] + [K^*][u^{\text{CAT}}] \]

where \([K^*]\) is defined herein as the local stiffness matrix, and is diagonal.

Remembering from Eq. (7) that \{u^{\text{CAP}}\} = \{\lambda^*_i\} [f] \text{ where } \lambda^*_i = (G_{ij} \text{ for } i \neq j \text{ and } 0 \text{ for } i = j) \text{ and with } \{f\} = -\{F\} = -\{S\}[u] \text{ Eq. (9) becomes}

\[ [[S] + [K^*][\lambda^*_i][S]] [u] = [K^*][u^{\text{CAP}}] \]

which can be solved numerically to obtain the solution. Omitting \([K^*][\lambda^*_i][S]\) in Eq. (10) results in a Winkler-like model, where the soil reaction acting on the pipeline is not affected by the soil response at different locations along the pipeline. The term \([K^*][\lambda^*_i][S]\) can thus be regarded as an additional term that takes account of continuum effects. This, however, does not mean that the solution obtained by omitting this term is the Winkler solution. This is due to the fact that the components of \([K^*]\) are different from those which will be constructed by commonly used subgrade coefficients, such as suggested by Vesic (1961).

To be able to solve Eq. (10), \{u^{\text{CAT}}\}, which is the green field displacement at the pipeline level, needs to be defined. The following section suggests a general form thereof.

**Table 1. Parameters for Eq. (12)**

<table>
<thead>
<tr>
<th>(n)</th>
<th>(\alpha)</th>
<th>(T = \text{Volume loss/}S_{\max})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.212</td>
<td>3.1061</td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
<td>2.5066 = \sqrt{2\pi}</td>
</tr>
<tr>
<td>1.5</td>
<td>0.79</td>
<td>2.2359</td>
</tr>
</tbody>
</table>

\(n\) \text{ is the volume loss parameter.}

**Soil Settlement Profile**

It is common practice to describe the vertical soil settlement, \(S_v\), due to tunneling using a Gaussian curve of the form:

\[ S_v = S_{\max} \exp \left[ -\frac{x^2}{2\sigma^2} \right] \]

where \(S_{\max}\) = maximum settlement; \(i\) = trough width parameter corresponding to the distance measured from the centerline of the tunnel to the inflection points of the curve. Unfortunately, in many cases the Gaussian curve is not satisfactory to accurately describe the soil settlement (e.g., Celestino et al. 2000; Jacobsz 2002). This lack of fit was also observed in the centrifuge experiments later referred to in this paper.

To obtain a better fit to the observed soil settlement, a modified Gaussian curve of the following form is suggested:

\[ S_v = \frac{n}{(n-1) + \exp \left[ \frac{1}{\alpha^2 - \frac{1}{2} + 1} \right]} S_{\max} \]

\(n\) \text{ is the volume loss parameter controlling the width of the profile; and } \alpha \text{ = parameter to ensure that } i \text{ remains the distance to the inflection point and therefore has the same definition as in Eq. (11). Table 1 summarizes the relation between } \alpha \text{ and } n \text{ given in Eq. (12), whereas Fig. 2 shows the effect of shape function parameter } n. \text{ Note, Eq. (12) becomes the Gaussian curve (Eq. (11)) when } n=1.\)

**Normalized Solution for the Maximum Bending Moments**

To enable a general solution corresponding to different soil and pipeline characteristics, the computed results were normalized.
Normalization covered all independent parameters and was used to describe bending moment and deflection response.

Fig. 3 shows the normalized sagging and hogging bending moments, $M^2/EI_S = f(K)$, in relation to a rigidity factor $K$, defined as $EI/Esr_0^3$ ($E_S$ = Young’s modulus of the soil). Fig. 3(a) shows the influence of the shape function (in terms of $n$) on the normalized pipeline bending moment in sagging and hogging when the pipeline embedment ratio, $Z/r_0$, is kept constant. Fig. 3(b) extends the information in Fig. 3(a) by showing the influence of $Z/r_0$ for shallower and deeper cases, when the shape function is kept constant. The results, in the range plotted in Figs. 3(a and b), were found to be practically independent of the ratio $r/r_0$ when $K$ was chosen as a nondimensional controlling parameter. The embedment depth effect is shown only for shape parameter $n=1$ (Gaussian curve), however similar behavior was observed for the other $n$ values. Although embedment depth, $Z$, may have a substantial impact on the bending moment of a pipeline due to its relation to $S_{max}$ and $i$ (which both change with depth), the normalized bending moment for a given $K$ is not affected significantly by $Z/r_0$ as shown in Fig. 3(b). Also shown in Fig. 3(b) are the infinitely flexible and rigid pipeline limits mentioned earlier, which provide perspective on the calculated values in respect of these limits.

If a normalized bending moment $M^2/EI_S = f(K)$ is considered rather than $M^2/EI_{S_{max}}$, then by definition it is equal to $f(K)K$. Fig. 4(a) shows this normalization for the Gaussian soil trough ($n=1$). A modified hyperbolic curve was fitted to the calculated values. From this curve, $f(K)$ was derived and is given and plotted in Fig. 4(b). The fitted curve corresponds very well to individually calculated points for $K$ in the higher range and fairly well to lower values of $K$. The fitted curve also exactly satisfies the behavior when $K$ tends to zero as the bending moment becomes solely a function of the soil displacement trough. The two limit cases plotted in Fig. 3(b) were obtained from the fitted curve under extreme values of $K=0$ and $K=\infty$. However, the rigid pipeline limit is not entirely accurate since it is based on a fitted curve over the range shown.

The effects of $S_{max}$ and $i$ on the bending moment are evident when expressing $M$ using the fitted expression for the case of $n=1$:

$$M = \frac{EIS_{max}}{r^2 + 0.55E^{1/3}Es^{-0.67}r_0^{-0.67}}$$  (13)

From this expression it is clear that the bending moment is always proportional to $S_{max}$. This is obviously related to the assumption of a linear system. On the other hand, the effect of $i$ is changing, having the most significant effect for flexible pipelines and a reduced effect for stiffer pipelines. As $EI/Esr_0$ approaches zero, the above-noted expression tends to that which will be derived from Eq. (1). It should be noted that Eq. (13) is suitable for cases where $0<K<100$ and should not generally be used for higher $K$ values since it was not fitted for that region.

It should be noted that all elastic analysis values presented in this paper assume Poisson’s ratio of 0.25 since it relates to sand for which experimental results are presented later. However, the effect of Poisson’s ratio was found to be limited—less than 1.5% between values of bending moment calculated with Poisson’s ratios ranging from 0.25 to 0.5.

**Contribution of the Relative Rigidity Factor**

From Fig. 3(a), it is evident that as the rigidity factor $K$ increases the different $n$ value curves tend closer to each other. The shape
function parameter \( n \) has more effect on the normalized hogging bending moment than the sagging bending moment. The maximum difference of the normalized sagging bending moment from the Gaussian curve is around 15\% for \( n=0.5 \) and 6\% for \( n=1.5 \), while for the hogging moment it is around 50\% for both \( n \) values. This behavior can directly be related to Fig. 2 in which it is seen that the shape parameter \( n \) affects the shoulders of the settlement trough more than the central section.

Further consideration of Fig. 3(a) reveals that when \( K \leq 0.1 \), the pipeline is practically following the soil settlement profile with the normalized bending moment basically coinciding with the case where \( K=0 \). As a result, the bending moment is mostly a function of the curvature of the soil [following Eq. (1)]. This is also evident from the normalized bending moment parameter, \( M_i/S_{max} \), which suggests that for this case the bending moment is proportional to \( S_{max}/K^2 \), which on its part is related to maximum curvature of the shape function (i.e., \( \partial^2 S_i/\partial x^2 \)) at \( x=0 \).

Different shape functions have different curvature. If the curvature is important, different shape functions result in different bending moments (under the same \( i \) and \( S_{max} \)). Conversely, if the bending moments are the same under different shape functions, the curvature is no longer important. Fig. 3(b) suggests that for \( K \geq 5 \), where the difference between normalized bending moments calculated from different shape functions becomes relatively small, the controlling parameter is no longer the curvature. It seems that a general shift from a demand for accurate estimation of the curvature (i.e., the shape function, \( S_{max} \) and \( i \)) to accurate estimation of just \( S_{max} \) and \( i \) occurs with increasing rigidity \( K \).

The above-noted observations provide designers with insight into the tradeoff between interaction analysis and green field measurements requirements. For example in cases where \( K \leq 0.1 \) significant field measurements need to be made to capture the curvature of the green field soil, while no pipe-soil interaction analysis is necessary as the pipeline practically follows the soil deflection. This suggests that, even if one undertakes an advanced finite element analysis, which takes account of complicated pipe-soil interaction aspects, the results may still be erroneous simply due to the difficulty in predicting the green field displacement using finite element analysis with conventional constitutive models (Addenbrookes et al. 1997). On the other hand, in cases where \( K \geq 5 \) pipe-soil interaction dominates and interaction analysis is required in order to obtain realistic bending moments. However, for this case less accurate green field representation is required since the behavior is less affected by the exact shape of the soil settlement trough. This tradeoff is shown schematically in Fig. 5.

It should be noted that in reality both \( i^2 \) and \( E_s \) (parameters in \( K \)) may change significantly with the geometrical configuration of the problem and soil nonlinearity. Subsequently, a pipeline may behave “flexibly” at one location but “stiff” at another, corresponding to its associated \( K \) value, i.e., in some cases it will be sufficient to disregard the pipe-soil interaction and simply force the pipeline to bend with the soil to obtain realistic bending moment values, while in other cases such a process will significantly overestimate the bending moment and interaction analysis is required.

### Normalized Moment Distribution along the Pipeline

Fig. 6 shows the normalized moment distribution and pipeline deflection for various rigidity factors and shape functions. It is evident that the pipeline rigidity and geometrical trough settlement parameters \( i \) and \( S_{max} \) are known, the rigidity factor \( K \) can be evaluated by estimating a value for the soil stiffness \( E_s \). Strictly speaking, the current elastic solution is based on Mindlin’s (1936) expressions and hence corresponds to homogeneous soil. In reality soil is rarely homogeneous. However, a fairly good approximation could be obtained for nonhomogeneous soil if \( E_s \) is taken as the soil stiffness at the pipeline embedment depth. This is supported by the fact that any loading of the soil resulting from the pipe-soil interaction is an internal loading to the continuum (of which the pipeline is a part), unlike external loading (e.g., foundation loading). Points far from any internal loading are unaware of its existence, unlike with external loading. As a result soil stiffness far from the pipeline may be regarded insignificant.

A good estimation of soil stiffness may not always be straightforward. However, even if the ratio of the true \( E_s \) and the estimated one is as large as 4, significant information on the behavior of the pipeline and subsequent field measurement-analysis tradeoff can still be obtained. This is evident from the shaded regions of Fig. 6 where a change of a factor of 4 in the value of \( K \) results in a significantly smaller change in normalized bending moment. This means that if the green field trough settlement is known and the soil stiffness can be approximated fairly well, these graphs may be used for design. However, an ad hoc estimate of \( E_s \) may not lead to a conservative design. In the following section a method of obtaining an upper approximation of bending moment through a systematic evaluation of \( E_s \) is suggested.

### Evaluation of Shear Strain for Soil Stiffness Determination

Using small strain stiffness, a linear elastic solution to the problem can be obtained under the assumption that the soil deflection at pipeline level is known. For a given green field soil settlement trough, any soil nonlinearity, whether resulting from pipe-soil interaction or from global shearing due to the tunnel, will reduce the maximum bending moment in the pipeline. This can be realized intuitively by observing that the pipeline would become less constrained to move with the green field soil. That means that the use of small strain stiffness will overestimate the true bending...
moment. Any additional reduction in the stiffness will still result in an upper approximation of bending moment, as long as the estimated soil stiffness is higher than the true one. This philosophy forms the basis of the proposed method by employing the elastic solution in an equivalent linear approach to the nonlinear problem.

The effect of shearing on soil stiffness is commonly represented by degradation curves, as illustrated in Fig. 7. These curves represent the degradation in the secant shear stiffness, \( G_{sec} \), with increasing engineering shear strain, \( \gamma \). A degradation relation may be extended to a general state of strain by considering quantities defined on the octahedral plane. A deviatoric strain space may be defined as follows:

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**Fig. 6. Normalized moment and displacement**

**Fig. 7. Stiffness degradation curve**
The square root of the second deviatoric strain invariant is equal to the magnitude of the above expression, \( \sqrt{J_2} = |\epsilon| \). In order to relate \( \gamma \) used in the degradation curve to the general strain state deviatoric strain, it may be evaluated as \( \gamma = 2[\epsilon] \).

Secant stiffness must be used in any equivalent linear analysis, and therefore, for such analysis, an underestimation of the shear strain will result in a stiffer soil compared to the reality (as long as the stiffness monotonically decreases with increasing shear strain as illustrated in Fig. 7). Shear strains around the pipeline would be greater than those in the green field, hence quantities of deviatoric strain as illustrated in Fig. 7 will result in a stiffer soil compared to the reality. The square root of the second deviatoric stain invariant is equal to \( \sqrt{\frac{1}{2} Tr(\epsilon:\epsilon)} \). The only components in the above-noted relation that are unknown directly from the shape function are the change of trough of soil displacement \( z \) at the tunnel level. This is also supported by Jacobsz (2002), who showed that volume loss decreases with depth. It was found that for \( z_R/i \) smaller than 0.4 it is possible for \( e_{xx} \) to be positive and hence violate the stated conditions. Nevertheless, reaching values of \( z_R/i \) below 0.4 is unlikely. In any case, \( 1/Ti \partial V L / \partial z \) is limited by \(-C S_{\text{max}} / i \) due to the condition that \( \partial S_{\text{max}} / \partial z > 0 \).

From the above considerations, Eq. (17) is reduced to

\[
\left| e_{xx} \right| \geq \frac{1}{2} \left[ -\frac{2\alpha \exp\left(\alpha x(z)\right)}{\left(1 + \exp\left(\alpha x(z)\right)\right)^2} \frac{S_{\text{max}}}{\mu} + \frac{\alpha x(z)}{\mu} \frac{S_{\text{max}}}{\mu} + \frac{2\alpha \exp\left(\alpha x(z)\right)}{\left(1 + \exp\left(\alpha x(z)\right)\right)^2} \frac{x S_{\text{max}}}{\mu} + \frac{x}{z_R \left(1 + \exp\left(\alpha x(z)\right)\right)^2} \right] \frac{S_{\text{max}}}{\mu} \frac{C}{i}
\]

where \( T = \text{volume loss parameter equal to } V L / S_{\text{max}} i \). In the case of undrained behavior \( \partial V L / \partial z \) is equal to zero and the complete expression can be solved. If the change of volume loss is such that it decreases with depth, the first component in Eq. (18) can be omitted to obtain a lower estimation \( |\gamma| \), but only if the resultant \( e_{xx} \) is negative. The experimental results used later indicate that this is indeed the case when comparing volume losses in the soil with that at the tunnel level. This is also supported by Jacobsz (2002), who showed that volume loss decreases with depth. It was found that for \( z_R/i \) smaller than 0.4 it is possible for \( e_{xx} \) to be positive and hence violate the stated conditions. Nevertheless, reaching values of \( z_R/i \) below 0.4 is unlikely. In any case, \( 1/Ti \partial V L / \partial z \) is limited by \(-C S_{\text{max}} / i \) due to the condition that \( \partial S_{\text{max}} / \partial z > 0 \).
Table 2. Parameters for Eq. (23)

<table>
<thead>
<tr>
<th>( n )</th>
<th>( A )</th>
<th>( B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.3385</td>
<td>0.1357</td>
</tr>
<tr>
<td>1.0</td>
<td>0.3824</td>
<td>0.2725</td>
</tr>
<tr>
<td>1.5</td>
<td>0.3957</td>
<td>0.2784</td>
</tr>
</tbody>
</table>

Strictly speaking \( e_{x,x} \) changes along the pipeline, and subsequently also the stiffness. Changing stiffness along the pipeline may be considered using a similar technique as suggested by Poulos (1979) for piles (i.e., that the Green’s function associating points \( i \) and \( j \) will be related to a mean value of the soil stiffness). Although such a method can easily be implemented in the numerical calculation, it is suggested to consider a constant reduced stiffness which will allow utilizing a single set of design graphs. An average value of the shear strain over an interval of \( 2.5i \) is suggested as the input parameter to find the reduced stiffness:

\[
\gamma_u = \frac{\int_0^{2.5i} |e_x|dx}{2.5i}
\]  

(20)

The interval of \( 5i \) (2.5i on either side of the tunnel centerline) corresponds approximately to the position of negligible settlement of the shape function. If only \( e_{x,x} \) is considered the following equality is true as long as \( e_{x,x} \) does not change sign in the interval of integration, which is also one of the earlier demands

\[
\gamma_u \geq \frac{\int_0^{2.5i} |e_{x,x}|dx}{2.5i} = \frac{\int_0^{2.5i} e_{x,x}dx}{2.5i}
\]  

(21)

Since \( e_{x,x} \) has a constant sign when \( Z_R/i \) is greater than about 0.4 the integral can be decomposed to obtain the following relation (for \( n=0.5 \)):

\[
\gamma_u \geq 0.3385 \frac{S_{max}}{i} + 0.6557 \cdot \frac{C}{Z_R} \frac{i}{n} S_{max} - 0.52 \cdot \frac{C}{Z_R} \frac{i}{m} S_{max}
\]

\[
= \left( 0.3385 + 0.1357 \frac{i}{Z_R} \cdot C \right) \frac{S_{max}}{i}
\]  

(22)

Terms I, II, and III correspond to the mean values of the three terms in Eq. (19). A similar form is obtained for the other shape functions; hence a general form is suggested:

\[
\gamma_u \geq \left( A + B \frac{i}{Z_R} \right) \frac{S_{max}}{i}
\]  

(23)

Table 3 summarizes \( A \) and \( B \) parameters according to the shape function:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( n=0.5 )</th>
<th>( n=1 )</th>
<th>( n=1.5 )</th>
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</thead>
<tbody>
<tr>
<td>( a )</td>
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<td>1.0</td>
<td>1.05</td>
</tr>
<tr>
<td>( b )</td>
<td>0.60</td>
<td>0.55</td>
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<tr>
<td>( \xi )</td>
<td>0.67</td>
<td>0.67</td>
<td>0.85</td>
</tr>
</tbody>
</table>

Values are valid for \( 0<K<100 \).

Design Procedure

To ensure a conservative evaluation of bending moment it is suggested that \( \gamma_u \) expressed by Eq. (23), is used as the input parameter for the linear equivalent upper approximation of soil stiffness. The following steps are needed to achieve an upper approximation:

1. Establish the green field soil displacement (i.e., \( n, i, S_{max} \)).
2. Calculate a lower approximation of a mean engineering strain, \( \gamma_u \), by using Eq. (23).
3. Estimate an appropriate small strain stiffness of the soil (e.g., Hardin and Black 1966; Seed and Idriss 1970; Jovicic and Coop 1997). Introduce \( \gamma_u \) into a degradation curve (e.g., Ishibashi and Zhang 1993; Tatsuoka et al. 1997) to obtain an upper approximation of the soil stiffness, \( Es \) (a constant Poisson’s ratio may be assumed for relating \( Es \) to \( G_{es} \)).
4. Calculate \( K \) using quantities from stages (1) and (3) and refer to Figs. 3(a and b) and 6 for upper approximations of the bending moment and pipeline deflection. For convenience modified hyperbolic curves similar to Fig. 4 were also fitted for the cases of \( n=0.5 \) and \( n=1.5 \). These may be referred to for an approximate value of the maximum sagging moment instead of scaling from Figs. 3(a and b). Table 3 provides values for the fitted curve of \( f(K) \) according to the following form:

\[
f(K) = \frac{a}{1+bK^2}
\]  

(24)

A step-by-step example of employing the proposed design method is shown in the Appendix.

Validation by Centrifuge Testing

The proposed method is used and evaluated against centrifuge test results. A series of tests were undertaken in the Cambridge centrifuge to investigate the effect of soft ground settlement on buried pipelines. A continuous model aluminum alloy pipeline of 15.875 mm outer diameter and 1.22 mm wall thickness was subjected to tunnel-induced ground movement in a series of centrifuge tests performed under 75 g acceleration, during which the boundary conditions of the problem were changed. A model tunnel was used to represent a 4.5-m-diam cavity on prototype scale (model diameter of 60 mm) and is similar to the one used by Jacobsz (2002). The tunnel comprises a hollow central brass mandrel over which a latex membrane is fitted. A known volume of water is filled in the annulus between the membrane and mandrel. By systematically extracting water, volume loss is induced at tunnel level, which manifests as ground movement throughout the model. The strong box was designed to minimize boundary effects following estimates of the zone of influence by Yeates (1984) and Attewell et al. (1986).
The tests were conducted in Leighton Buzzard Fraction E silica sand ($d_{50}=0.142$ mm, $e_{\text{max}}=0.97$, and $e_{\text{min}}=0.64$; Lee 2001) with a void ratio of approximately 0.68.

To enable quantifying the effect of tunnel-induced ground movement on the model pipeline, measurements were made as follows: (1) LVDT and laser measurement to measure green field surface settlement outside the expected influence of the pipeline (green field), (2) LVDTs with extensometers to measure subsoil settlement at a level corresponding to the invert level of the pipeline, but outside the expected influence of the pipeline, (3) bending moments in the pipeline by means of strain gauges. More details about the model and the experimental setup can be found in Vorster (2002).

For each geometrical configuration, subsoil displacement at pipeline level and bending moment were extracted for tunnel volume losses of 0.5, 1, 1.5, 2, and 4%. For each volume loss a shape function was fitted to the measured subsoil settlement to obtain appropriate values of $S_{\text{max}}$ and $i$. It was found that for these cases $n=0.5$ gave the best fit to the measured data. Following the four steps suggested earlier for upper approximation of bending moment, calculated values were obtained for each pipeline at the above-mentioned volume losses. In step (3) of the suggested method the expressions of Seed and Idriss (1970), and Ishibashi and Zhang (1993) were used to obtain the maximum shear stiffness and its degradation with shear strain, respectively. Similar maximum stiffness values to Seed and Idriss (1970) were also obtained by using Jovicic and Coop (1997).

Fig. 8 shows the centrifuge model during model preparation with a schematic of the experiment, explaining the geometric cases used for validation of the proposed design method. Fig. 9 shows the comparison between the normalized calculated values and the experimental ones. The results are identified by the embedment depth, referred to as SL (shallow embedment, 11% of tunnel depth), IL (intermediate embedment depth, 24% of tunnel depth), and DL (36% of tunnel depth), and according to the associated volume loss. Although the actual bending moment increases with volume loss, the normalized values, presented in Fig. 9, decrease. This is due to the simultaneous increase of $S_{\text{max}}$ and decrease of $i$ with increasing volume loss.

As shown in Fig. 9, all the results fall below the equality line, $M_{\text{Exp}}=M_{\text{Calc}}$, indicating that the method is an upper approximation as argued. A trend is observed indicating that at smaller strains, associated with smaller volume losses, the method approximates the experimental readings more successfully than at larger volume losses. This is consistent with the fact that for smaller volume losses less nonlinearity exists in the soil, and hence the elastic solution is more appropriate. It should be noted that significant nonlinearity occurs even at the smaller volume losses; according to the suggested method (which underestimates nonlinearity) a 60% reduction in stiffness relative to $G_{\text{max}}$ occurs already at a volume loss of 0.5%.

The benefit of the suggested design method can be illustrated by comparing the bending moments derived by various approaches of design. If a simplified method of design is applied where the pipeline is “forced” to bend with the soil, the calculated values overestimate the measured ones by up to 3 times for 0.5% volume loss and up to 20 times for 4.0% volume loss. If the small
strain stiffness is to be used without a reduction, the calculated values overestimate the measured values up to tenfold (for 4.0% volume loss). If an elastic Young’s modulus for the sand is taken in the range of 30–50 MPa (commonly used value for loose to dense sand in engineering), calculated values overestimate the measured bending moments up to five- to sevenfold (for 4.0% volume loss). Note, the arbitrary choice of a stiffness between 30 and 50 MPa results in seemingly better approximation for other volume losses, but does not promise an upper approximation as the suggested method does, and therefore might not be conservative. On the other hand, values calculated from the suggested upper approximation method appear not to exceed 3 times the measured values for the pipeline system in question. It is important to note that the method does not take into account all parts of the deviatoric strain space and therefore cannot give an exact estimation of bending moments, but only upper approximation.

The difference between the upper approximation and the measured data for the intermediate (IL) and deep embedment cases (DL) are in the same range, while it is significantly more in the case of shallow embedment (SL). It seems that in order for the result to be in the same order of deviation from the upper approximation, greater nonlinearity must be taken into account. The upper approximation method introduces only nonlinearity that originates from the tunnel and is apparent in the green field. It is possible that for this case the low relative pullout failure load, associated with shallow depth, accounts for the significant additional nonlinearity. It is expected that nonlinearity due to pipe-soil interaction also occurs at greater depth; however, the relative contribution in relation to global shearing becomes smaller with depth.

Fig. 10 shows the calculated values of K corresponding to the calculated normalized bending moment in Fig. 9. It is evident that K increases with volume loss and changes in order of magnitude due to changes in i and Es. A pipeline that is classified as “flexible” at low volume loss may therefore be considered “stiff” at higher volume loss. Furthermore, for a given tunnel depth, the trend apparent from the calculation is that a certain pipeline would behave “stiffer” as embedment increases.

Summary and Conclusion

A method was presented for obtaining an upper approximation of bending moment for pipelines affected by tunnel-induced ground movement. The method is an equivalent linear approach which utilizes a closed form solution of subsoil displacement to derive deviatoric strain. The derived deviatoric strain is smaller than its true value and hence results in an equivalent linear stiffness exceeding the true one, which leads to an upper approximation of bending moment. In order to validate the suggested procedure, centrifuge experiment results were compared with the calculated upper approximation of bending moment. The comparison showed that the method indeed results in an upper approximation. It is, however, recommended that the proposed design procedure be validated in future also by means of full-scale field testing.

A general solution for the elastic problem is presented in a normalized fashion. The general solution corresponds to a subsoil settlement presented by a modified Gaussian curve, for which the Gaussian curve is a special case. A rigidity factor, K, was recognized to represent the key interaction factor. For example in cases where K ≤ 0.1, significant investigation into the field settlement trough (field measurements or applying case histories of similar problems) needs to be made, while no pipe-soil interaction analysis is necessary as the pipeline practically follows the soil deflection. This suggests that even if one undertakes an advanced finite element analysis which takes account of complicated pipe-soil interaction aspects the results may still be erroneous simply due to slightly inaccurate modeling of the green field response. On the other hand, in cases where K ≥ 5, pipe-soil interaction is dominant and interaction analysis is therefore in order to ensure a realistic estimate of bending moment. However, for this case less field measurements are required since the behavior is less affected by the exact shape of the soil settlement trough.

Although the method includes nonlinearity effects due to the green field displacement, any nonlinearity resulting from pipe-soil interaction is absent. This results in overestimation of the sagging moments. Advanced mechanisms such as relative uplift failure and gapping between the pipeline and soil, which would contribute additionally to nonlinear soil behavior, were observed in some of the centrifuge tests and are reported in Vorster et al. (2005). An attempt to introduce some of these aspects into the analyses, by means of local plasticity, was made by Klar et al. (2005b).

It should also be noted that the suggested method does not consider the effect of pipeline joints allowing rotation or axial movement.

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Appendix. Design Example

To illustrate the use of the proposed upper approximation design method for calculating the maximum sagging moment, consider a hypothetical case where a cast iron pipeline with 0.4 m outside radius ($EI=105,000$ kN m$^2$) and axis located at a depth of 1.5 m below ground surface, is affected by the excavation of a 1.5-m diam tunnel with its axis situated at a depth of 5 m below ground level. The tunnel is excavated in dry sand (e=0.67, $\gamma_s=16$ kN/m$^3$) with rounded grains inducing a volume loss of 5% of the face area with a trough width parameter, i=2.6 m. It is
assumed that a Gaussian curve fits the green field settlement profile at pipeline level.

Step 1: Establish the green field soil displacement. Using a modified Gaussian curve the parameters in Table 4 apply for describing the expected green field settlement profile.

Step 2: Calculate a lower approximation of the mean engineering shear strain, $\gamma_a$. From Eq. (23) and Table 2 and with $C=0.325$ (Mair et al. 1993) and $Z_R=3.5$ m

$$\gamma_a \geq \left(0.3824 + 0.2725 \frac{i}{Z_R}ight) \frac{S_{\text{max}}}{i} \geq 2.344 \times 10^{-3}$$ (25)

Step 3a: Estimate an approximate maximum stiffness of the soil at pipeline level. From Hardin and Black (1966) the maximum shear modulus, $G_{\text{max}}$, is estimated as 40,916 kPa.

Step 3b: Introduce $\gamma_a$ into a stiffness degradation curve to find an upper approximation of $E_s$. Using Ishibashi and Zhang (1993)

$$G_{\text{sec}}/G_{\text{max}} = 0.14$$ (26)

giving $G=5728$ kPa.

To find $E_s$ a constant Poisson’s value of $v=0.25$ is applied giving $E_s=14,320$ kPa.

Step 4: Calculate the pipe-soil rigidity factor, $K$, and the maximum sagging moment, $M$

$$K = \frac{E_l}{i^3 r_g E_s} = 1.04$$ (27)

$K$ is greater than 0.1 and smaller than 5, which means that both curvature and interaction analyses may be necessary to obtain a better approximation of the bending moment provided by this design method, if required.

Using Eq. (24) and values from Table 3 the normalized bending moment, $f(K)$, is given as

$$f(K) = \frac{1}{1+0.55K}^{0.67} = 0.64$$ (28)

This indicates that the normalized bending moment would have been overestimated by 36% had the pipeline been forced to follow the green field settlement profile.

From Fig. 3 the normalized bending moment is also

$$f(K) = \frac{M_i^2}{E_l S_{\text{max}}}$$ (29)

From the above equation $M=134.9$ kN m inducing a bending strain of 514 $\mu$m, which is considered high.

From Fig. 6 for $n=1$

$$S_i/S_{\text{max}} = 0.88$$ (30)

which gives $S_{\text{sec}}=12.0$ mm.

In terms of ease-of-use, the proposed design method may be compared to the method suggested by Attewell et al. (1986). It may be of interest to the reader that, if the method by Attewell et al. (1986) is used to estimate the maximum sagging moment based on the same $E_s$, $M$ is 116.5 kN m. Although this value is slightly lower (~14%) than the one estimated by the method proposed in this paper, it is not known whether it is conservative. Attewell et al. (1986) used a Winkler system to estimate the solution by applying Vesic’s (1961) spring coefficient. Klar et al. (2004) showed that the use of Vesic’s (1961) spring coefficient would not generally give identical results to that of a continuum solution for settlement induced loads and suggested a revised spring coefficient to use in the Winkler system to correct this.

Using the revised spring in a Winkler system to evaluate this example results in a near-identical bending moment of 133.8 kN m

### Notation

The following symbols are used in this paper:

$A,B =$ parameters defining a minimum for $\gamma_a$;

$a,b,\xi =$ parameters describing the fitted function of the normalized bending moment, $f(K)$, for different shape functions;

$C =$ change of $i$ with depth $-(d\lambda/dz)$;

$D =$ pipeline bending stiffness;

$E_s =$ Young’s modulus of the soil;

$e =$ void ratio;

$E =$ force vector representing soil loading on the pipeline;

$f =$ force vector acting on the soil medium $-\{E\}$;

$G_{i,j} =$ Green’s function which defines the elastic soil continuum displacement at point $i$ due to unit loading at point $j$;

$G_{\text{max}} =$ small strain shear modulus;

$G_{\text{sec}} =$ secant shear modulus;

$i =$ distance to the inflection point of the green field settlement trough;

$J_{2D} =$ second deviatoric strain invariant;

$K =$ relative pipe-soil rigidity factor;

$[K] =$ local soil stiffness matrix;

$k =$ coefficient of subgrade reaction;

$M =$ bending moment in the pipeline;

$M_{\text{Calc}} =$ normalized bending moment calculated by the suggested design method;

$M_{\text{Exp}} =$ normalized bending moment based on data measured in the Cambridge centrifuge and employing a shape function with $n=0.5$ and $\alpha=0.212$.

$M_{\text{max}} =$ maximum bending moment in the pipeline (sagging);

$n =$ shape function parameter;

$r_g =$ radius of the pipeline;

$[S] =$ stiffness matrix of the pipeline;

$S_{\text{max}} =$ maximum settlement in the green field;

$S_p =$ pipeline settlement;

$S_v =$ vertical settlement in the green field;

$T =$ volume loss parameter equal to $VL/S_{\text{max}} i$;

$u =$ pipeline displacement;

$u^C =$ soil continuum displacement;

$u^{CA} =$ additional displacement at a point due to forces acting at other places;

$u^{CAP} =$ additional displacement resulting from forces of pipe-soil interaction;

$u^{CAT} =$ additional displacement resulting from the tunnel;

$u^{CL} =$ local continuum displacement defined as the displacement at a point due to its own loading;

$VL =$ volume loss at the tunnel;

$Z =$ pipeline embedment depth below ground level;

$Z_R =$ distance between the pipeline and tunnel axis;

### Table 4. Parameters for the modified Gaussian curve in the design example

<table>
<thead>
<tr>
<th>$n$</th>
<th>$a$</th>
<th>$S_{\text{max}}$</th>
<th>$i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
<td>13.6 mm</td>
<td>2.6 m</td>
</tr>
</tbody>
</table>

$^a$Table 2.

$^b$Table 1.
References


